

Nonlinear Forecasting analysis of inflation-deflation patterns of an active caldera (Campi Flegrei, Italy)

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ABSTRACT

The ground level in Pozzuoli, Italy, at the center of the Campi Flegrei caldera, was monitored by tide gauges between 1970 and 1976 and then continuously since 1982. Tide gauges offer a long record of a variable that is believed to be related to the activity of an underlying shallow magma chamber. Previous work suggests that the dynamics of the Campi Flegrei system, as reconstructed from the tide gauge record, is chaotic and low dimensional. According to this suggestion, in spite of the complexity of the system, at a time scale of days the ground motion is driven by a deterministic mechanism with few degrees of freedom; however, the interactions of the system may never be describable in full detail. Our new analysis of the tide gauge record from January 1987 to June 1989, using Nonlinear Forecasting, confirms low-dimensional chaos in the ground elevation record at Campi Flegrei and suggests that Nonlinear Forecasting could be a useful tool in volcanic surveillance.

INTRODUCTION

The Campi Flegrei volcanic area is historically known for the continuous slow pulsations of the ground (e.g., Lyell, 1830, p. 326–339; Lirer et al., 1987) that closely accompany other symptoms of volcanic activity. Lyell (1853, p. 507–519) provided an elegant discussion of ancient ground-level changes in this region, relative to sea level. His deduction was based in part on past submersion documented by molluscan borings found up to 6 m above the base of marble columns in a marketplace built in the second century B.C. in the town of Pozzuoli, at the center of the Campi Flegrei caldera (Fig. 1). In 1538 an uplift of ~7 m in 2 days preceded the last eruption in the area, which had been undergoing uplift since 1502 (Dvorak and Gasparini, 1991). Recent uplift events, of ~2 yr duration, took place in 1970–1971 (1.56 m near the center of the caldera) and in 1982–1984 (1.85 m, also near the center of the caldera) and were accompanied by increased fumarolic and seismic activity. Both the local population and the scientific community had to contend with the possibility of a volcanic eruption in a densely populated area (~400 000 inhabitants within a caldera ~12 km in diameter).

Substantial stratigraphic, petrological, and geophysical evidence suggests that a shallow magma chamber, the roof of which is probably 3.5–4 km below the ground surface, is responsible for the volcanic activity in Campi Flegrei (Barberi et al., 1984; Lirer et al., 1987; De Natale et al., 1991; Luongo and Scandone, 1991). Ground movement is the primary signal of volcanic activity in the area (Corrado and Luongo, 1981; Barberi et al., 1984; Osservatorio Vesuviano, 1985–1990; Berrino and Corrado, 1991). In spite of the progress in modeling the Campi Flegrei ground deformations as a consequence of the activity of a shallow magma chamber (e.g., see De Natale et al., 1991), an understanding of the cause and effect is incomplete and may ultimately be unattainable because of the complexity of the system and the multitude of the possible interactions.

Cortini et al. (1989) proposed that the Campi Flegrei volcanic and subvolcanic system could be considered as a thermodynamic black box for which only one state variable of the system is known, the ground level at Pozzuoli. The Campi Flegrei area is unstable on

time scales ranging from days to thousands of years; at the shortest time scale, the system shows a macroscopic response to tidal cycles; that is, to differential gravitational forces of only ~1 ppm. In this perspective, Cortini et al. (1991) concluded that the Campi Flegrei system, at time scales between days and thousands of years, is far

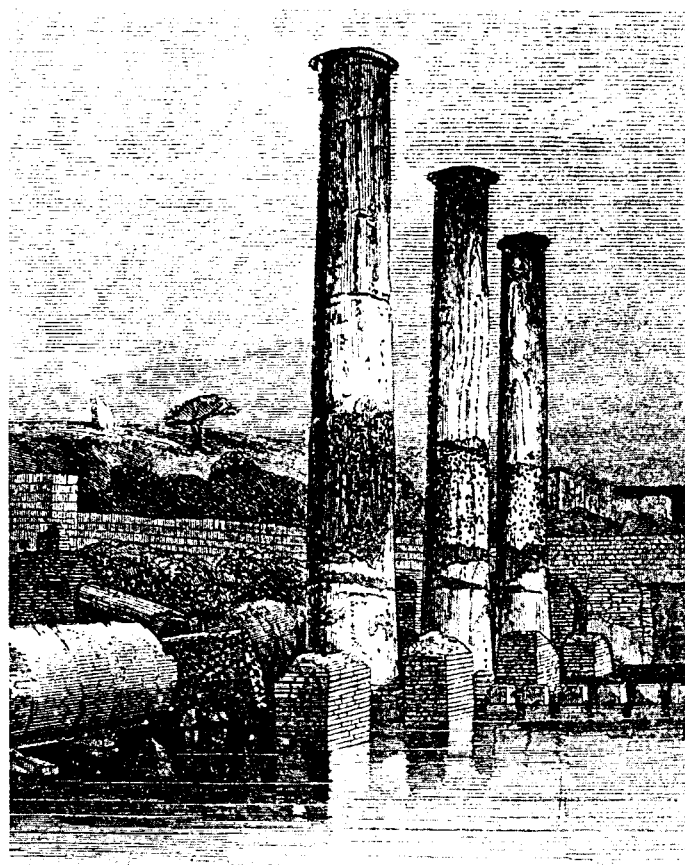


Figure 1. Illustration of marble columns in second century B.C. marketplace (previously thought to be temple of Serapis) at Pozzuoli, Italy. Frontispiece from Lyell (1853). *Lithodomus* borings indicate submergence of the columns to ~6 m above their bases during previous deflation cycle of Campi Flegrei.

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from thermodynamic equilibrium, and that its dynamics (reconstructed by the daily tide gauge record) may be chaotic. However, when observed at a time scale of tens of thousands of years, the Campi Flegrei system is approaching thermodynamic equilibrium, i.e., a stable fixed state that in phase space (see next section) is described as a point attractor. On the basis of differences between the frequency spectra of the ground motion in the 1971–76 and 1987–89 periods, Cortini et al. (1991) proposed that the high-frequency dynamics of Campi Flegrei underwent a critical transition between phase locking (with tidal forces) and low-dimensional chaos, probably in coincidence with the uplift crisis of 1982.

Three nonlinear forecasting methods are reviewed by Farmer and Sidorowich (1988) and Casdagli (1989). The forecasting method we use in this paper has been described and used on nongeological data sets by Sugihara and May (1990) and is implemented by Dynamical Systems Inc.¹ (Schaffer and Tidd, 1990). We use it to test the hypothesis of chaos in the Campi Flegrei tide gauge record and to explore the possibilities for surveillance offered by this method.

NONLINEAR FORECASTING

Nonlinear Forecasting is a recent development in the theory of dynamical systems. Imagine two identical deterministic systems starting from slightly different initial conditions. If, as time passes by, the two systems diverge exponentially, then their dynamics is said to be “chaotic.” This characteristic is called “sensitive dependence on initial conditions” and explains why a chaotic system can be predictable in the short term but is inherently unpredictable in the long term, even though the forces that drive it may be perfectly known (Cvitanovic, 1984; Schaffer et al., 1988).

The temporal evolution of a physical-chemical system is called its dynamics and can be described by its state variables. A state of a system can be represented as a point in a conventional space (state, or phase space) whose n coordinate axes $x_1(t)$, $x_2(t)$, . . . , $x_n(t)$ each represent a state variable; that is, an independent parameter that is relevant to the description of the system. When the system changes, so does the position of its representative point in the phase space. The dynamics of a system is represented by a trajectory in phase space, and study of the geometrical properties of that trajectory can provide information about the system’s dynamics. In the study of natural systems, one generally has only a time series $x(t)$, but a phase portrait can be reconstructed by plotting $x(t)$ vs. $x(t + T)$ vs. $x(t + 2T)$ vs. . . . $x[t + (n + 1)T]$, where T is an appropriate time delay, or lag. The dynamics thus portrayed is said to be embedded in an n -dimensional space. The geometry of the reconstructed phase portrait can be studied and quantified.

The difference between a random, or noisy, dynamics and a chaotic dynamics is in the magnitude of n , the number of state variables required to fully describe the system. The number of state variables required to describe random noise tends to infinity, whereas chaotic dynamics can be described by as few as three state variables. This difference is the basis of one method used to identify chaos, in which one measures the dimension of the phase-space region visited by the system; this is termed the correlation dimension (Grassberger and Procaccia, 1983). This approach can give quantitatively misleading results if the data set is small or if the data are treated incorrectly; moreover, the choice of the intervals used to calculate the slope of the correlation integrals is always somewhat arbitrary (e.g., Schaffer et al., 1988).

Nonlinear Forecasting is a stronger method for diagnosing chaos. In this method, first the time series is lagged and embedded

in an n -dimensional phase space, as described above. A mapping of the visited phase-space volume onto itself is then constructed as follows. A small volume δV_n in phase space, visited at time t_n , is then linked with a new volume δV_{n+1} , visited by the system at time $t_n + \delta t$. This procedure is repeated for every point visited by the system in phase space. This mapping, or library, of the phase-space volume is then used to formulate predictions about the future evolution of the system (Sugihara and May, 1990). The difference between a random, or noisy, dynamics and a chaotic dynamics can now be described as follows. A noisy dynamics is perturbed by many different causes and can move from a small phase-space volume δV_n to any nearby volume. In contrast, a chaotic dynamics, because it is deterministic, must move in a well-defined direction. Because of the sensitivity to initial conditions, δV_{n+1} will be larger than δV_n and, for k large enough, δV_{n+k} will cover the entire visited volume. This permits the formulation of predictions for $k < k_0$, where k_0 is some threshold value.

The dynamics of natural systems are always obscured by some amount of noise, which derives at least in part from the fact that numbers fed to computers can have only a limited number of digits; that is, they are approximations. Other sources include experimental error and, because natural systems are never isolated, the interference of other systems. Noise tends to decrease the goodness of fit measured by the correlation coefficients between the predicted and the observed values (Fig. 2). If noise is too large, or if the number of the state variables is too great, a chaotic dynamics that may underlie a system cannot be identified, and prediction becomes impossible.

Figure 2 shows the results of Nonlinear Forecasting applied to a well-known chaotic dynamics (that of the logistic map for $k = 3.7$; May, 1976). Multiplicative Gaussian noise is added to the data series analyzed in the upper curve by replacing each of the 500 data points x by $x(1 + z)$, where z is a number randomly selected from a Gaussian distribution with a mean value of zero and with $\sigma = 0.025$. The correlation coefficient r^2 between the predicted and the observed values is plotted as a function of the prediction interval. The shape of these curves is typical of chaotic dynamics that are predictable only on a limited time horizon.

The initial 20% of the data are used to produce the first prediction, which is compared with the following point. The library is updated for as long as later points are analyzed; thus, the last prediction is based on a library five times as large as the first one.

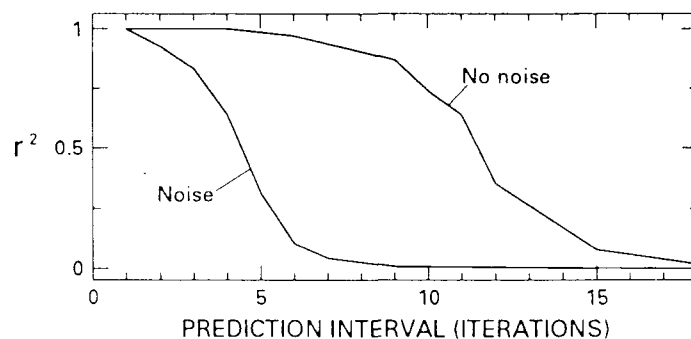


Figure 2. Upper curve illustrates Nonlinear Forecasting applied to sample chaotic data series 500 points long (logistic map for $k = 3.7$) (see May, 1976). Correlation coefficient r^2 between predicted and observed values is plotted as a function of prediction interval δt (which, in this example, is number of iterations of logistic map). Lower curve is produced by contamination of chaotic data series with Gaussian noise as described in text. Shape of both curves is typical of chaotic dynamics that is predictable on a short term (because of underlying determinism) but unpredictable on longer terms (because of sensitivity to initial conditions).

¹Any use of trade names is for descriptive purposes only and does not imply endorsement by the U.S. Geological Survey.

Throughout this paper we set the embedding dimension $D_e = 2$, the embedding delay $T = 1$, and the prediction parameter that sets the size of δV_n , $\epsilon = 0.005$, because these values maximize r^2 . Addition of noise decreases the length of the time interval over which reliable predictions can be made because it increases the rate at which information about the system is lost (Wales, 1991).

DATA AND ANALYSIS

We analyzed the numerical daily tide gauge record $x(t_n)$ from January 1, 1987, to June 27, 1989 (Fig. 3). We calculated the first differences of the data to analyze the daily series $\delta x_n = x(t_n) - x(t_{n+1})$ (i.e., the daily elevation changes). The reason for calculating first differences is as follows. The magnitude of $x(t_n)$ is a few metres, and that of δx_n is a few millimetres. If the series δx_n were completely random, then the best prediction one could make would be that $x(t_{n+1}) = x(t_n)$, and the error on the predicted value would be of the order of 10^{-3} . For increasing prediction intervals, the predicted values slowly diverge from those of the analyzed random walk, and the correlation coefficient between the predicted values and the observed values slowly decreases as the prediction interval increases, as for a chaotic dynamics (Fig. 2), although the dynamics is a random walk. Taking the first differences of the data preserves its variations, which in this example are random. The error on the prediction is now of the same order as the signal. In this example, the correlation coefficient r^2 between the predicted values and the observed values will be very low and independent of the prediction interval δt , and a plot of r^2 as a function of δt will be very different from that in Figure 2. This procedure reveals whether the high-frequency variations of the ground elevation are chaotic and readily distinguishes a chaotic from a random dynamics.

Figures 4 and 5 illustrate how the predicted values compare with the observed values for a prediction interval $\delta t = 1$ day; perfect predictions align on the line of unit slope in Figure 4. The correlation coefficient r^2 between the data predicted for the following day and those actually observed on the following day is ~ 0.77 . The points

align on vertical lines because the resolution of the tide gauge record is limited to 1 mm; Nonlinear Forecasting, in contrast, does not recognize this limitation. Figure 6 shows how the linear correlation coefficient r^2 varies as a function of the prediction interval. The results of the analysis for variations of ground motion in Campi Flegrei, plotted in Figures 4–6, are typical of a chaotic dynamics, possibly blurred by some noise (Sugihara and May, 1990). As noted above, we set the embedding dimension $D_e = 2$ and the embedding delay $T = 1$. We find that our predictions vary by $<10\%$ with large variations ($D_e \leq 5$, $T \leq 20$) in these parameters. Thus, our choice of values for these parameters is not critical to our results.

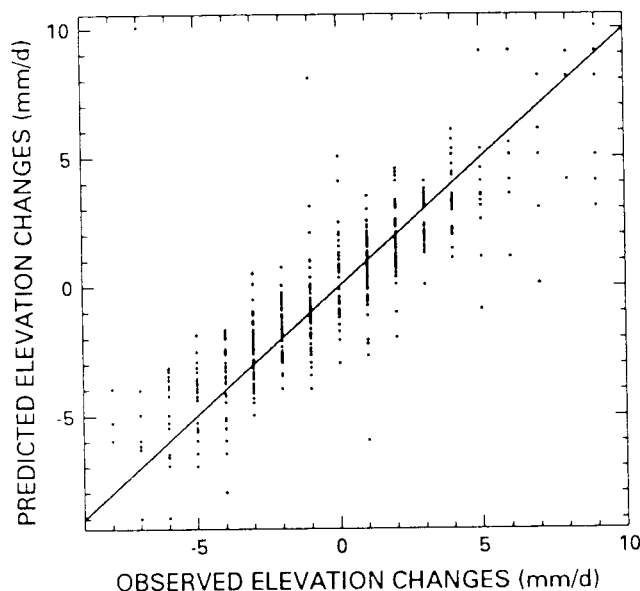


Figure 4. Nonlinear Forecasting analysis of data shown in Figure 3 after calculating first differences. Predicted daily elevation changes are plotted as a function of observed elevation changes for prediction interval $\delta t = 1$ day (i.e., for following day). Linear correlation coefficient r^2 between predicted and observed values is ~ 0.77 .

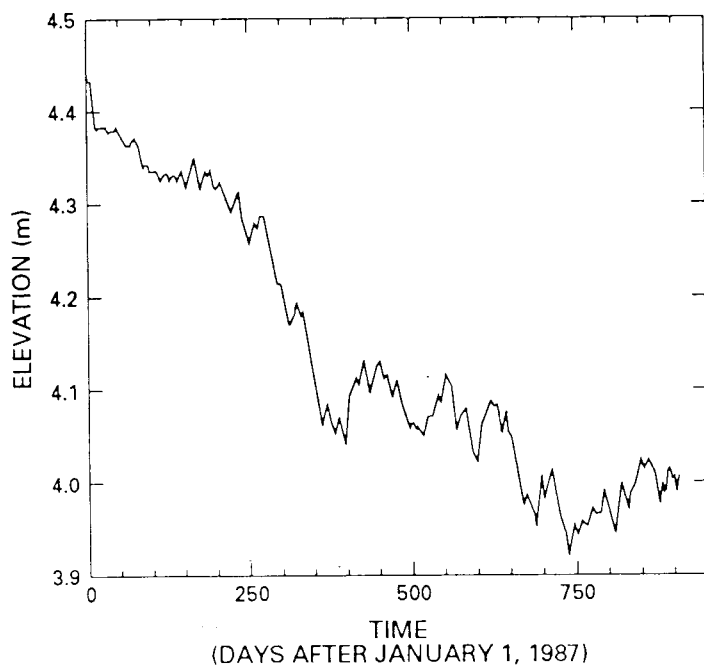


Figure 3. Daily ground elevation at harbor of Pozzuoli (January 1, 1987, to June 27, 1989), near center of Campi Flegrei caldera, as measured by tide gauges (Corrado and Luongo, 1981; Berrino and Corrado, 1991; data from Osservatorio Vesuviano).

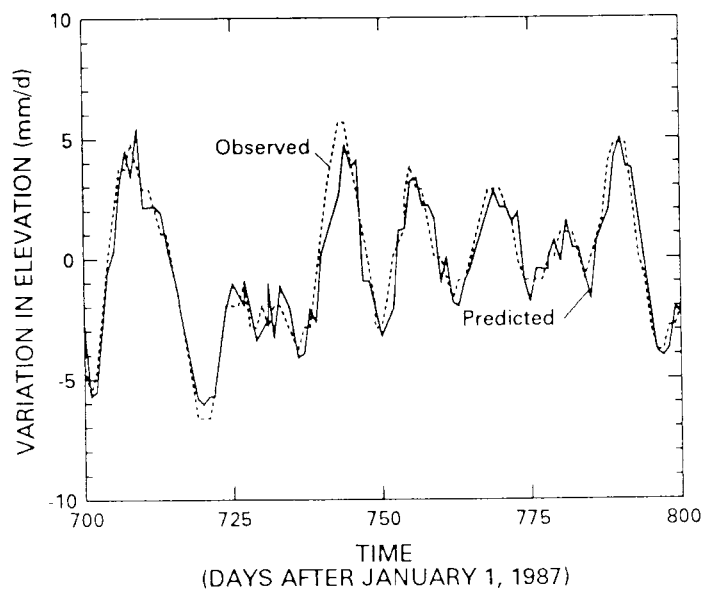


Figure 5. Nonlinear Forecasting analysis of data shown in Figure 3 after calculating first differences. Observed and predicted data series (prediction interval $\delta t = 1$ day) are plotted as a function of time.

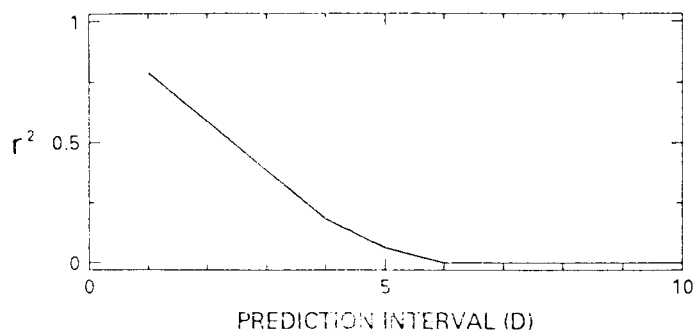


Figure 6. Nonlinear Forecasting analysis of data shown in Figure 3 after calculating first differences. Linear correlation coefficient r^2 between predicted and observed values, calculated as in Figure 4, is plotted as a function of prediction interval. Note similarity to chaotic results plotted in Figure 2.

The analysis presented above is limited to the small data set available at present. The time period analyzed is one of overall ground sinking (Fig. 3). The complete time series, including time periods of overall uplift, should be analyzed; unfortunately, these data were not available to us at the time of this study. A detailed comparison between the post-1970 and the post-1984 data is also necessary to fully test the hypothesis, suggested by Cortini et al. (1991), that the two periods are characterized by different dynamical regimes.

CONCLUSIONS

Nonlinear Forecasting of the daily elevation changes demonstrates that the ground motion in the Campi Flegrei caldera is chaotic. The evidence for chaos provided by Nonlinear Forecasting is stronger than that provided by the Grassberger-Procaccia method because of the uncertainties in that method (Farmer and Sidorowich, 1988). Nonlinear Forecasting requires very little manipulation of the data, only calculating first differences, whereas analysis of the short-term dynamics of Campi Flegrei using the Grassberger-Procaccia method requires subjective detrending. Our results show that the variations of the ground elevation can probably be forecast on a time scale of a few days. Why a complex system such as a magma chamber, interacting with many external factors, may on some time scales have a relatively simple and predictable dynamics, is a question that is not addressed here.

This study includes a time period <3 yr long. Given a longer record, the Campi Flegrei system may be intrinsically more predictable over time scales longer than 1–2 days. The agreement between the predictions of Nonlinear Forecasting and the ground-level data, on a time scale of days, suggests that this technique should be more thoroughly explored because it potentially could be of great help during a possible future activity crisis in Campi Flegrei, a very high risk volcanic area. The use of this method in volcanic surveillance to forecast future events immediately prior to an eruption is a formidable and unprecedented problem that is beyond the scope of this paper.

We note that vertical ground movements are recorded almost exclusively on calderas located by the sea, such as Campi Flegrei and Rabaul in Papua, New Guinea (McKee et al., 1984), where elevation changes are very conspicuous. In fact, it was Pozzuoli fishermen who, in 1970, noticed that the ground was rising. Such large ground motions also may be common in many active volcanic areas away from such a convenient datum as the surface of the sea, and an effort should be made to detect and record these ground motions as well.

ACKNOWLEDGMENTS

Early versions of this paper benefited from review by USGS colleagues R. R. Charpentier and H. R. Shaw, by Alberto Malinverno of Lamont-Doherty Geological Observatory of Columbia University, and by discussions in the USGS Fractals/Chaos Study Group in Denver, Colorado. Supported in part by the Gruppo Nazionale per la Vulcanologia of the Italian Research Council.

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Manuscript received August 4, 1992

Revised manuscript received November 6, 1992

Manuscript accepted November 11, 1992